

Fermilab

-p Note #376

Interim Note on Computer Model of Super-  
Conducting Coax

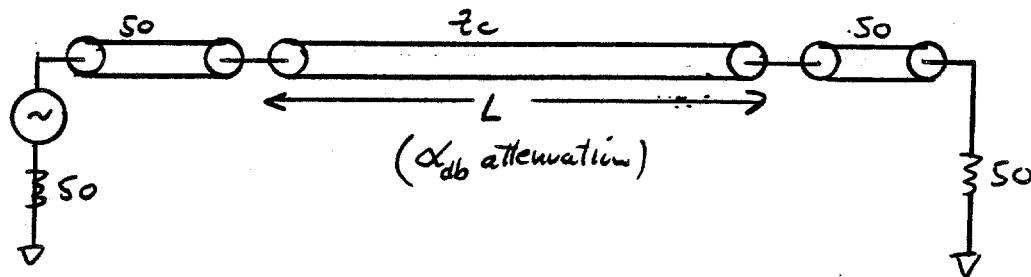
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3/15/84

Interim Note on Computer Model of Superconducting Coax

The following discussion is a simplistic analysis of the return loss (i.e reflected power) observed on the tests of the Furukawa coax last November. See Figure 1.

If we assume a simple model of the form



and assume that the network analyzer is an ideal  $50\ \Omega$  system, and that the superconducting coax can be represented by a coax of length  $L$ , characteristic impedance  $z_c$ , and an attenuation  $\alpha_{db}$  db per km, then we can write down an algorithm to represent the reflected voltage:

$$V_R = V_F \left( \frac{50 - z_c}{50 + z_c} \right) \left( 1 - e^{-2\alpha L} e^{i2\beta L} \right) \quad (1)$$

If we represent  $e^{i2\beta L}$  by

$$e^{i2\beta L} = \cos(2\beta L) + i \sin(2\beta L) \quad (2)$$

then  $\frac{V_R}{V_F} = \left( \frac{50 - z_c}{50 + z_c} \right) \left( 1 - e^{-2\alpha L} \cos(2\beta L) - i e^{-2\alpha L} \sin(2\beta L) \right)$  (3)

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and

$$\left| \frac{V_R}{V_F} \right| = \left( \frac{50 - Z_C}{50 + Z_C} \right) \left[ (1 - e^{-2\alpha L} \cos(2\beta L))^2 + (e^{-2\alpha L} \sin(2\beta L))^2 \right]^{1/2} \quad (4)$$

The return loss is then in db:

$$(RL)_{db} = 20 \log \left[ \frac{50 - Z_C}{50 + Z_C} \right] + 20 \log \left[ (1 - e^{-2\alpha L} \cos(2\beta L))^2 + (e^{-2\alpha L} \sin(2\beta L))^2 \right]^{1/2} \quad (5)$$

In particular the minimum and maximum return losses are

$$(RL_{min})_{db} = 20 \log \left[ \frac{50 - Z_C}{50 + Z_C} \right] + 20 \log \left[ 1 - e^{-2\alpha L} \right] \quad (6)$$

$$(RL_{max})_{db} = 20 \log \left[ \frac{50 - Z_C}{50 + Z_C} \right] + 20 \log \left[ 1 + e^{-2\alpha L} \right] \quad (7)$$

Hence

$$(RL_{max})_{db} - (RL_{min})_{db} = 20 \log \left[ \frac{1 + e^{-2\alpha L}}{1 - e^{-2\alpha L}} \right] \quad (8)$$

which is independent of  $Z_C$ .

The measurements indicate that  $(RL_{max})_{db} \approx -20 \text{ db}$

and

$$(RL_{min})_{db} \approx -40 \text{ db}$$

which leads to

$$20 \log \left[ \frac{1 + e^{-2\alpha L}}{1 - e^{-2\alpha L}} \right] = 20 \text{ db} \quad (9a)$$

$$\frac{1 + e^{-2\alpha L}}{1 - e^{-2\alpha L}} = 10 \quad (9b)$$

$$\text{or } \alpha = -\frac{\ln \frac{9}{11}}{2L} = -3.04 \times 10^{-4} \text{ nepers/meter} \quad (9c)$$

for  $L = 330$  meters. This implies

$$\alpha_{db} = 20 \log e^{-\alpha L} = \frac{20}{\ln 10} \ln e^{-1000\alpha} = \frac{20,000\alpha}{\ln 10} \quad (10a)$$

$$= 8686\alpha = -2.64 \text{ db per km.} \quad (10b)$$

This is not in agreement with attenuation measurements which yield approximately 1 db per km. at 1.5 GHz.

$$\text{Since } (RL_{max})_{db} \sim -20 \text{ db and } 20 \log \left( 1 + e^{-2\alpha L} \right) = +5.19 \text{ db}$$

$$\text{then } 20 \log \left| \frac{50 - Z_C}{50 + Z_C} \right| = -25.19 \text{ db} \quad (11)$$

$$\text{or } \left| \frac{50 - Z_C}{50 + Z_C} \right| = 0.055$$

$$\text{hence } Z_C = 50 \left( \frac{1 - 0.055}{1 + 0.055} \right) = 44.8 \text{ ohms} \quad (12a)$$

$$\text{or } Z_C = 50 \left( \frac{1 + 0.055}{1 - 0.055} \right) = 55.8 \text{ ohms} \quad (12b)$$

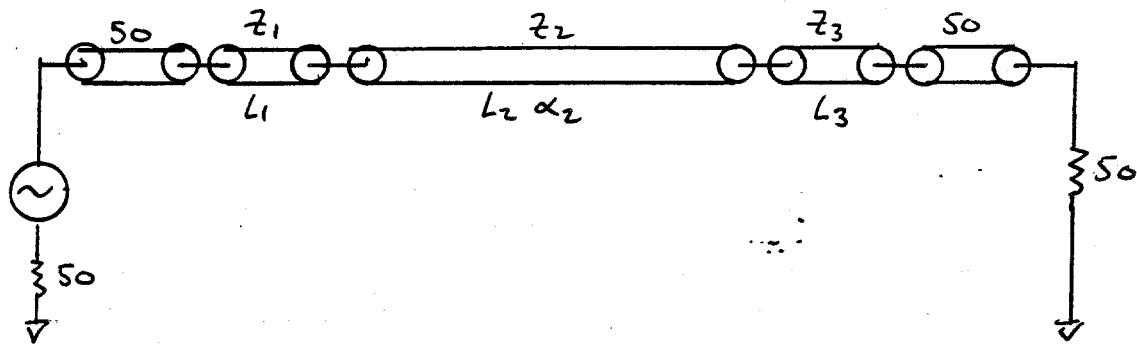
TDR measurements imply that  $Z_C = 48 \Omega$

The return loss is plotted in Fig 2 for  $L = 330 \text{ m}$

$$\alpha = 2.64 \text{ db/km}$$

$$Z_C = 44.8 \text{ ohms.}$$

Suppose we now add two cryogenic transition lines which are lossless, but have a characteristic impedance  $z_1$  and  $z_3$ , and lengths  $L_1$  and  $L_3$ :



In this case, again neglecting multiple reflections, and assuming that the propagation velocity is the same everywhere, we have:

$$V_R = V_F \left\{ \frac{50 - z_1}{50 + z_1} + \frac{z_1 - z_2}{z_1 + z_2} e^{i2\beta L_1} + \frac{z_2 - z_3}{z_2 + z_3} e^{-2\alpha L_2} e^{i2\beta(L_1+L_2)} + \frac{z_3 - 50}{z_3 + 50} e^{-2\alpha L_2} e^{i2\beta(L_1+L_2+L_3)} \right\} \quad (13)$$

splitting this into real and imaginary components we have

$$\left| \frac{V_R}{V_F} \right| = \left[ (A + B \cos(2\beta L_1) + C e^{-\alpha L_2} \cos[2\beta(L_1+L_2)] + D e^{-\alpha L_2} \cos[2\beta(L_1+L_2+L_3)])^2 + (B \sin(2\beta L_1) + C e^{-\alpha L_2} \sin[2\beta(L_1+L_2)] + D e^{-\alpha L_2} \sin[2\beta(L_1+L_2+L_3)])^2 \right]^{1/2} \quad (14)$$

$$\therefore (\text{Return Loss})_{db} = 20 \log \left| \frac{V_R}{V_F} \right| \quad (15)$$

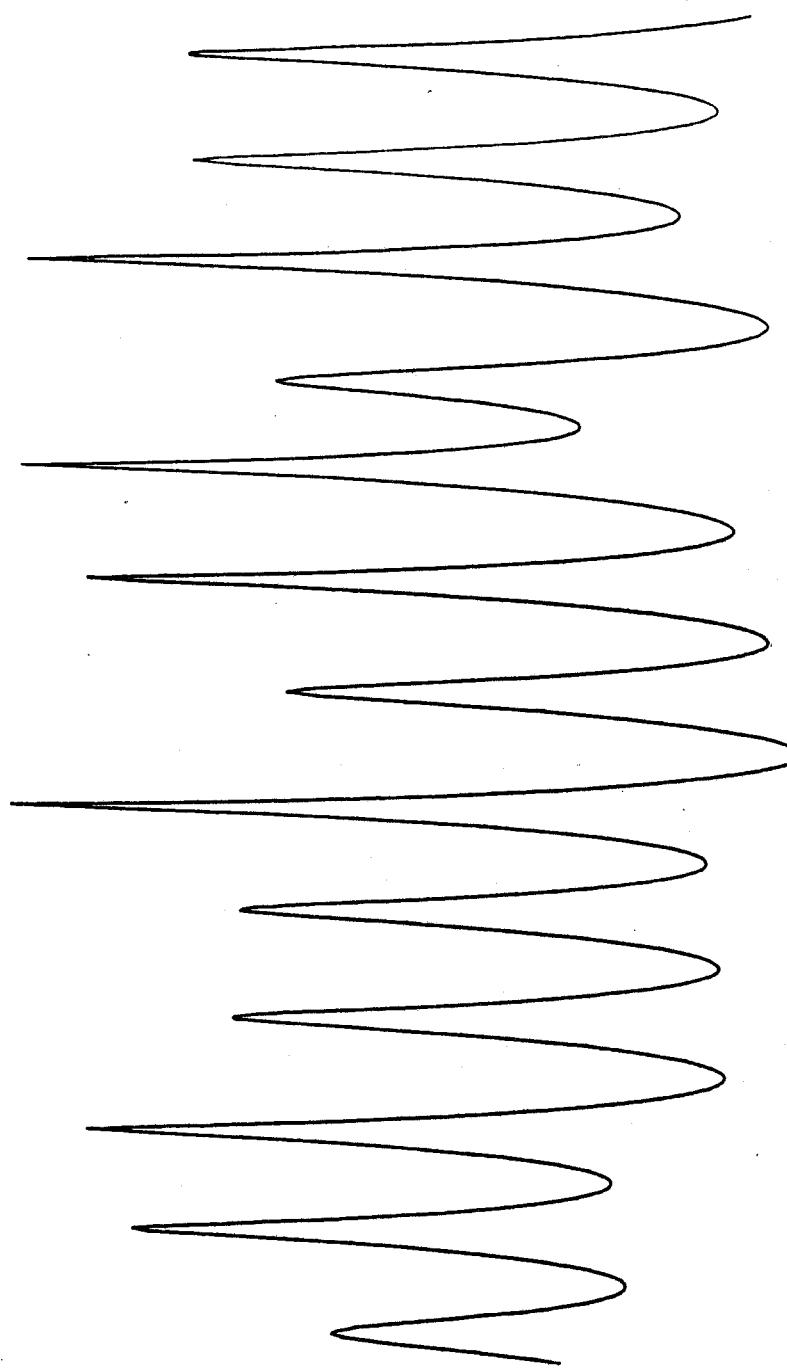
Equation (15) is plotted in Figures 3, 4, and 5 for the following parameters

|               | <u>Fig 3</u> | <u>Fig 4</u> | <u>Fig 5</u> |
|---------------|--------------|--------------|--------------|
| $L_1, L_3$    | 30m          | 30m          | 30m          |
| $L_2$         | 330m         | 330m         | 330m         |
| $\alpha_{db}$ | 2.64 db/km   | 2.64 db/km   | 2.64 db/km   |
| $Z_1, Z_3$    | 49.52        | 44.852       | 46.52        |
| $Z_2$         | 44.85        | 48.52        | 48.52        |

Hence it is apparent that the large modulation of the return loss can be caused either by an impedance mismatch of the superconducting coax or of the coaxial structure at the ends.

More analysis needs to be done.

FIGURE II. COAX RETURN LOSS (DB)



330 m superconducting coax  
feedlines included

file = PERIOD3/UN = 95686 11/18/83

FIGURE 1

